BINOMIAL THEOREM

Topic 1:

Binomial Theorem for positive integral index

Binomial expression

An algebraic expression consisting of two terms with +ve or -ve sign beth them, is called binomial ~~theorem~~ expression.

e.g $(a+2b)$, $\left(\frac{p}{x^2} - \frac{q}{x^4}\right)$ $\left(3x - \frac{2}{y}\right)$

Pascal's Triangle.

In earlier classes, we have already studied that

i) $(a+b)^0 = 1$
ii) $(a+b)^2 = a^2+2ab+b^2$
iii) $(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$
iv) $(a+b)^1 = a+b$
v) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Pascal's Triangle With The Help of Combination

To find the expansion of the binomial for any power without writing all the rows of the pascal's triangle that come before the row of the desired index. We can use a rule which is based on combination

We know that ${}^nC_r = \frac{n!}{r!(n-r)!}$ $(0 \le r \le n)$ and n is non-negative integer. Here ${}^nC_0 = {}^nC_n = 1$

Write the Pascal triangle as given below

Index — Coefficient

4
1 2 — ${}^0C_0$ (−1)
2 3 — ${}^1C_0$ (−1) ${}^1C_1$ (−1)
3 — ${}^2C_0$ (−1) ${}^2C_1$ (−2) ${}^2C_2$ (−1)
4 — ${}^3C_0$ (−1) ${}^3C_1$ (−3) ${}^3C_2$ (−3) ${}^3C_3$ (−1)
5 — ${}^4C_0$ (−1) ${}^4C_1$ (−4) ${}^4C_2$ (−4) ${}^4C_3$ (−4) ${}^4C_4$ (−1)
${}^5C_0$ (−1) ${}^5C_1$ (−5) ${}^5C_2$ (−5 ${}^5C_3$ (−5) ${}^5C_4$ (−5) ${}^5C_5$

For $n=7$ the row would be

${}^7C_0, {}^7C_1, {}^7C_2, {}^7C_3, {}^7C_4, {}^7C_5, {}^7C_6, {}^7C_7$

Binomial Theorem for any Positive integer

Theorem: If a and b are two real numbers, then for any positive integer n, we have

$$(a+b)^n = {}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 \ldots\ldots + {}^nC_n a^0 b^n$$

$$\rightarrow (a+b)^n = \sum_{r=0}^{n} {}^nC_r a^{n-r} b^r$$

Important formulae

i) ${}^nC_r = \frac{n!}{r!(n-r)!}$ $(0 \le r \le n)$

ii) ${}^nC_0 = {}^nC_n = 1$, ${}^nC_1 = n$

iii) ${}^nC_r = {}^nC_{n-r}$

iv) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

v) ${}^nC_x = {}^nC_y \iff x = y$ or $x + y = n$

vi) $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$

vii) ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots {}^nC_n = 2^n$

viii) ${}^nC_0 + {}^nC_2 + \ldots = {}^nC_1 + {}^nC_3 + \ldots = 2^{n-1}$